Sequences and series

Dear students,

Now we will study one of the important part of number theory. From the lower stds we are in connection with series, and we have to face the questions like "What is the next number in the given series ? e.g. 2, 4, 6, 8 ….

Sequence is an ordered collection of objects in which repetitions are allowed like a set it contains numbers. (also called elements or terms). Sequences are the basis for series. One way to specify a sequence is to list the elements. E.g. 1, 3, 5, 7, first four odd numbers.

There are many important integer sequences. E.g. 2, 4, 6, 8, … all even +ve integers. In that first important part is prime numbers. (We have already studied about them.)

We also can form the different sequences using natural numbers such as …..

$n + d$, n^2, … etc. giving values as n = 1, 2, 3, ….. increasing natural numbers. We will get the following series ---

i. $n + d \rightarrow$ 5, 9, 13, 17, 21, 25, ….. n = 1, 2, … & d = 4
ii. $n^2 \rightarrow$ 1, 4, 9, 16, 25, 36, …..
iii. $n^2 + 1 \rightarrow$ 2, 5, 10, 17, 26, 37, ……..
iv. $n^2 - 1 \rightarrow$ 0, 3, 8, 15, 24, 35, ……..
v. $n^3 \Rightarrow$ 1, 8, 27, 64, 125, ……
vi. $n^2 + n \Rightarrow$ 2, 6, 12, 20, 30, …..
vii. $n^2 - n \Rightarrow$ 0, 2, 6, 12, 21, …….
viii. $N^3 + 1 \Rightarrow$ 2, 9, 27, 65, 126, ………
ix. $N^3 - 1 \Rightarrow$ 0, 7, 26, 63, 124, …….
x. $N^3 \pm n^2 \Rightarrow$ 2, 10, 36, 80, 130 or 0, 4, 18, 48, ….. etc.

With different combinations of addition and subtraction of natural numbers we can form different sequences.

Other than this, we different imagination we can form variety of series also. as ….

(i) Whose spellings do not contain a certain letter or alphabets. The 'e' ban numbers (numbers which do not contain 'e'.)
The series is written as → 2, 4, 6, 30, 32, 34, 36, 40, 42, ……
(ii) The numbers of letters in the number's English spelling -
3, 3, 5, 4, 4, 3, 5, 5, 4, 3, 6, 6, 8, 2, …
(One, Two, Three, Four, Five, Six, …..)

While studying the number theory, some series are very important, so they are studied separately.

1. Arithmetic Series → a, $a + d$, $a + 2d$, $a + 3d$, ……
Here series starts with some constant ('a') and increases or decreases by common difference ('d')
e.g.
(i) 3, 7, 11, 15, 19, ….. here $a = 3$ & $d = +4$
(ii) 10, 8, 6, 4, 2, 0, -2, -4, …. $a = 10$ & $d = -2$
2. A Geometric Series → a series written by product i.e. by multiplying the previous term by a constant (called the common ratio)
e.g.
(i) $1, \frac{1}{2}, \frac{1}{4}, \frac{1}{8}, \frac{1}{16}$, ….. here $a = 1$ & $r = \frac{1}{2}$
(ii) a, ar, ar^2, ar^3 …….

3. Harmonic Series → e.g. 1, $\frac{1}{2}, \frac{1}{3}, \frac{1}{4}, \frac{1}{5}$, ….. it is alternating numbers in the series.

4. Arithmetic Geometric sequence → This is a generalization of geo series, which has, coefficients of common ratio of the terms in an arithmetic series. e.g. $\frac{3}{1}, \frac{5}{2}, \frac{7}{4}, \frac{9}{8}, \frac{11}{16}$ ….. general term is $\frac{3+2n}{2^n}$

5. An alternating series → is a series where terms alternate signs.
   e.g. 1, $-\frac{1}{2}, \frac{1}{3}, -\frac{1}{4}, \frac{1}{5}, \frac{1}{6}$ ….. a general term is $(-1)^{n+1} \times \frac{1}{n}$

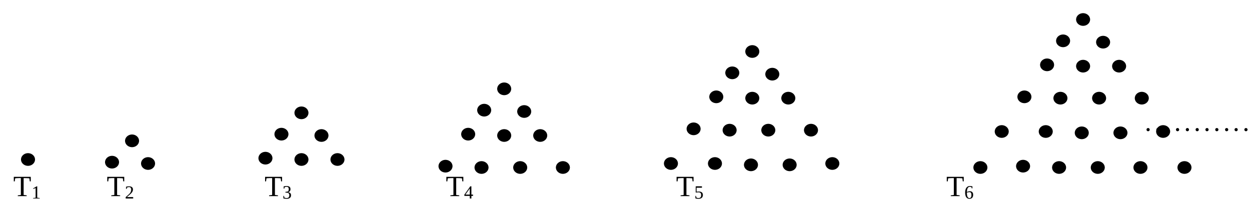
6. 'P' series → Can be written in any one of two different forms
   e.g. $\frac{1}{1^2}, \frac{1}{2^2}, \frac{1}{3^2}, \frac{1}{4^2}$, …….. i.e. $\frac{1}{n^r}$ n = 1 to ∞ & r > < 1

7. A Telescoping Series → $b_n - b_{n-1}$ where n = 1 to 100 and so on.

Thus we will come across more types of interesting series also. While writing different series we study different types of numbers, such as prime numbers, composite numbers, odd numbers, even numbers or irrational numbers also.

In this pattern I will like to add two types of numbers (I) Triangular numbers and (II) Square numbers.

I) Triangular numbers → Triangular numbers are known by the arrangement of dots.



1 3 6 10 15 21 …………..

$Tn = \frac{n\,(n+1) \quad 2}{2}$

If we observe these numbers we will see that sum of two consecutive numbers is a perfect square

e.g. 0, 1, 3, 6, 10, 15, 21, 36, 45, 55, 66, 78, 91. . . . . .

$0 + 1 = 1 = 1^2$ $6 + 10 = 16 = 4^2$

$1 + 3 = 4 = 2^2$ $10 + 15 = 25 = 5^2$ $45 + 55 = 100 = 10^2$

$3 + 6 = 9 = 3^2$ $66 + 78 = 144 = 12^2$ etc.

Important property of these numbers can be stated as their digital root is always 1, 3, 6 or 9

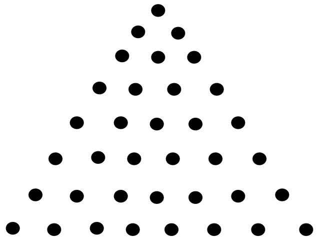
e.g. 1 = 9 x 0 + 1 10 = 9 x 1 + 1 21 = 9 x 2 + 3

3 = 9 x 0 + 3 15 = 9 x 1 + 6 55 = 9 x 6 + 1

II) Square numbers which can be stated as square of any no. for e.g. $2^2, 3^2, 4^2$, ...

III) But Square Triangular Numbers are numbers which can be shown is both triangular as well as square numbers.

e.g. 36 can be shown as



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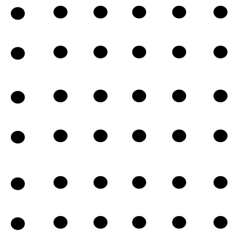

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There are infinite number of square numbers. First few are
1, 36, 1225, ...

Students, we will stop here, because the numbers are not ending but we have a time limit.

Thank you